Exercise 21

Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$$f(x) = 3x - 8$$

Solution

Calculate the derivative of $f(x)$ using the definition.

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{[3(x+h) - 8] - (3x - 8)}{h} \\ &= \lim_{h \to 0} \frac{(3x + 3h - 8) - 3x + 8}{h} \\ &= \lim_{h \to 0} \frac{3h}{h} \\ &= \lim_{h \to 0} 3 \\ &= 3 \end{aligned}$$

The domain of $f(x)$ is $\{x \mid -\infty < x < \infty\}$, and the domain of $f'(x)$ is $\{x \mid -\infty < x < \infty\}$. $f(x)$ and $f'(x)$ are polynomials, so any number can be plugged into them.